7-4 Remainder and Factor Theorems

Solve.
$x^3 + 4x^2 - 15x - 18 = 0$
If $x - 3$ is a factor.

$$\begin{array}{r|rrrr} 3 & 1 & 4 & -15 & -18 \\ & & 3 & 21 & 18 \\ \hline & 1 & 7 & 6 & 0 \end{array}$$

$x^2 + 7x + 6 = 0$
$(x+6)(x+1) = 0$

$\{+3, -6, -1\}$

Solve.
$x^3 + 7x^2 + 2x - 40 = 0$
If $x - 2$ is a factor.

$\{-5, -4, 2\}$

$$\begin{array}{r|rrrr} 2 & 1 & 7 & 2 & -40 \\ & & 2 & 18 & 40 \\ \hline & 1 & 9 & 20 & 0 \end{array}$$

$x^2 + 9x + 20 = 0$
$(x+5)(x+4)$

Solve.
$x^3 - 2x^2 + 9x - 18 = 0$
If $x - 2$ is a factor.

$\{\pm 3i, 2\}$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & 9 & -18 \\ & & 2 & 0 & 18 \\ \hline & 1 & 0 & 9 & 0 \end{array}$$

$x^2 + 9 = 0$
$x = \pm 3i$

When it is a factor, what can you say about the remainder?

Is it a factor?

$f(x) = x^3 + x^2 + 3x + 3$

Is $x + 2$ a factor? No

$$\begin{array}{r|cccc} -2 & 1 & 1 & 3 & 3 \\ & & -2 & 2 & -10 \\ \hline & 1 & -1 & 5 & -7 \end{array}$$

$f(x) = x^3 + x^2 + 3x + 3$

Find $f(-2)$.

$= (-2)^3 + (-2)^2 + 3(-2) + 3$

$= -7$

$f(x) = x^3 + x^2 + 3x + 3$

Is $x + 3$ a factor?

No

$f(-3) = -24$

$f(x) = x^3 + x^2 + 3x + 3$
Is $x + 1$ a factor? YES

$$\begin{array}{c|cccc} -1 & 1 & 1 & 3 & 3 \\ & \downarrow & -1 & 0 & -3 \\ \hline & 1 & 0 & 3 & 0 \end{array}$$

Find k such that
$2x^4 + x^3 + 5x^2 - 6x + k$ $\div$ $(x + 2)$ has a remainder of 5.

$$\begin{array}{c|cccc} -2 & 2 & 1 & 5 & -6 & k \\ & & -4 & 6 & -22 & 56 \\ \hline & 2 & -3 & 11 & -28 & 56+k \end{array}$$

$56 + k = 5$
$k = -51$

$2x^4 + x^3 + 5x^2 - 6x + k$
Find k such that $x + 2$ is a factor.

$k + 56 = 0$
$k = -56$

<u>Remainder Theorem</u> (summary)
The remainder of $f(x)$ $\div$ $(x - a)$ is $f(a)$.

<u>Factor Theorem</u> (summary)
The binomial $(x - a)$ is a factor of $f(x)$ iff $f(a) = 0$.

HW
p368-369
13-17, 21-27, 31-35 all odds